SOFTENING THE HABITATS
SUSTAINABLE INNOVATION IN MINIMAL MASS STRUCTURES AND LIGHTWEIGHT ARCHITECTURES

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Powerful Tools for Formfinding, Statics and Patterning of Pneumatic Structures

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Abstract

Formfinding is needed to create a pneumatically feasible surface. Therefore, the force density approach is extended with additional constraints. The additional constraints are one or several chambers, which can be loaded by different internal pressure values or even simpler by their volumes. By this method forms can be generated without any limitations concerning the boundary conditions. A harmonically stressed surface is guaranteed. Reinforcements by cables, belts, etc. can be considered already in the form finding procedure.

Statics starts with the definition of the material properties. In general, we define for textile membranes: warp- and weft stiffness, and, if available, crimp- and shear stiffness. We must fix an internal (operating) pressure and now the non-deformed geometry of the finite membrane elements, we can say the patterns, can be calculated. Load case calculations can be performed now by 3 different modes: Constant inner pressure (p=constant), constant volume (V=constant), constant product of inner pressure and volume (p·V=constant, gas law of Boyle-Mariotte) or even the general gas equation (p·V/T=constant). Sometimes belts or cables are used to reinforce the structure. We can define those elements by either fixing them onto the membrane or leave them completely free on the membrane. The pneumatically stressed membrane surface is usually attached to a bending stiff primary structure. The common interaction between the pneumatically stressed membrane and the primary structure is considered by a hybrid model.

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We show that the disregarding of the common interaction leads to 'wrong' results and should be not applicated. We try to improve the computer models in many aspects: as an example, we would like to mention the wind-loads; in our calculation the wind changes its direction and size with respect to the deformation, we apply so-called non-conservative loads. Very important in membrane engineering is to avoid ponding problems, so we developed tools to show the flow of the water onto the surface.

After the form is found the patterning can be done. We show how to divide the form in several substructures and how to pattern the different parts individually. Automatic tools for the optimization of the widths can be used. Fast methods for compensation, seam allowances, welding marks, help to produce high quality patterns. We also provide so-called evaluation numbers for the patterning to help the membrane engineers to get perfect patterns without any wrinkles.

Keywords: Pneumatic structures, Biogas plants, Lightweight structures, Formfinding, Statics, Patterning, Gas law, Hybrid structures, Membranes, Foils, Force density, Optimization

1 Formfinding

The Formfinding procedures of mechanically and pneumatically stressed structures is very similar at first view. In both cases prestress values for the membranes are set; for pneumatically structures we need additionally also an internal pressure or a specific volume (which is finally caused by an internal pressure). This procedure has one disadvantage concerning the pneumatic structures: the geometry cannot be controlled as it is the result of the Formfinding method. But in some cases, the geometry should be as simple as possible (as a geometrical function) in order to end up with simple patterns, etc. Here we must make sure that these surfaces are pneumatically feasible. Only a few geometrical functions such as spheres, cylinders, torus shaped forms are useful.

This is the reason why we distinguish between model generation by a Formfinding procedure and model generation by geometrical forms being pneumatically feasible.

1.1 Model generation with Formfinding procedure

In the following we describe the Formfinding procedure of pneumatically stressed structures. It is known that a Formfinding procedure solves the ‘inverse’ problem. ‘Inverse’ is related to Statics where the geometry and its elements including size, stiffness, etc. are given and forces, stresses and (small) deflections are searched. The Formfinding procedure in contrast inputs stress values for the elements and ends up with a balanced geometry of a pneumatically prestressed surface. As usual Formfinding procedure of pneumatic structures works as follows: a membrane field (with its material directions) is inputted with ‘desired’ prestress-values in warp and weft directions and volume or an internal pressure. In Figure 1 we have a typical example. The red mesh shows the membrane field and the blue line is the polygon where the membrane is finally fixed to.

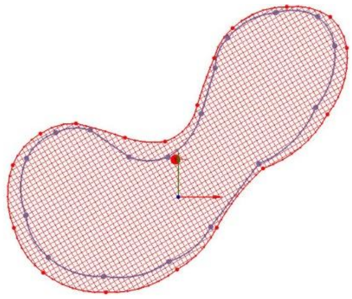

Figure 1: Top view of a meshed boundary as input for a Formfinding

The result of the method (here the force-density approach was used) is the balanced shape in 3D with very uniform stress values. If the same input stresses are used in warp and weft direction we end up with a minimal surface.

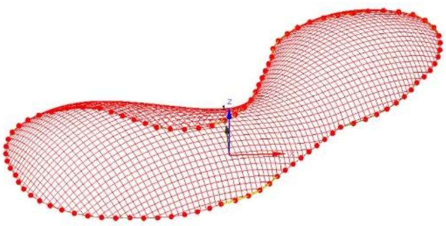

Figure 2: Formfinding result

This example consists of only the membrane surface and the membrane is simply fixed by a closed line onto a plane. This line can be assumed as absolutely fixed.

Theoretical background: The Formfinding of pneumatic chambers has its basics in the well-known Force-Density Method ([1], [2] and [3]). The Force-Density Method creates a linear system of equations for the form-finding procedure by defining the ratio between Force *S* and stressed length *l* to be known. Hereby the nonlinear equations of the equilibrium change to a linear system.

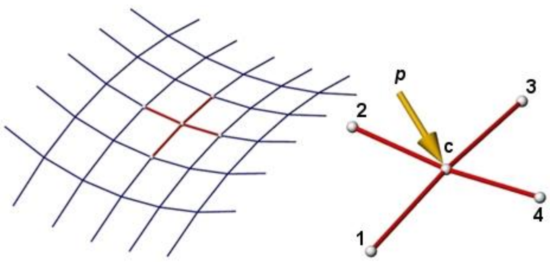

Figure 3: Four cables in point C

In order to clarify these facts Figure 3 shows a point *C* which is connected by cables to 4 points (*1,2,3,4*). The nonlinear equations of the equilibrium in the point *C* are as follows, where the external load vector can be expressed $\boldsymbol{p}^t = (p_x \quad p_y \quad p_z)$.

$$\begin{array}{lllll} (x_c - x_1)\frac{S_1}{l_1} + & (x_c - x_2)\frac{S_2}{l_2} + & (x_c - x_3)\frac{S_3}{l_3} + & (x_c - x_4)\frac{S_4}{l_4} & = & p_x \\ (y_c - y_1)\frac{S_1}{l_1} + & (y_c - y_2)\frac{S_2}{l_2} + & (y_c - y_3)\frac{S_3}{l_3} + & (y_c - y_4)\frac{S_4}{l_4} & = & p_y \\ (z_c - z_1)\frac{S_1}{l_1} + & (z_c - z_2)\frac{S_2}{l_2} + & (z_c - z_3)\frac{S_3}{l_3} + & (z_c - z_4)\frac{S_4}{l_4} & = & p_z \end{array} \qquad (1)$$

These equations become linear by assuming known force-densities, e.g. $q_1 = \frac{S_1}{l_1}$, and analogue for q_2, q_3 and q_4. The force-density equations are as follows:

$$\begin{array}{llllll} (x_c - x_1)q_1 + & (x_c - x_2)q_2 + & (x_c - x_3)q_3 + & (x_c - x_4)q_4 & = & p_x \\ (y_c - y_1)q_1 + & (y_c - y_2)q_2 + & (y_c - y_3)q_3 + & (y_c - y_4)q_4 & = & p_y \\ (z_c - z_1)q_1 + & (z_c - z_2)q_2 + & (z_c - z_3)q_3 + & (z_c - z_4)q_4 & = & p_z \end{array} \qquad (2)$$

The coordinates of the point *C* are the solution of these linear equations. In the following step we want to write the system above by considering m neighbors in the point *C*:

$$\begin{array}{lll} \sum_{i=1}^{m}(x_i - x_c)q_i - p_x & = & 0 \\ \sum_{i=1}^{m}(y_i - y_c)q_i - p_y & = & 0 \\ \sum_{i=1}^{m}(z_i - z_c)q_i - p_z & = & 0 \end{array} \qquad (3)$$

The energy which belongs to the system (1) can be written as (see also [4], [5]).

$$\prod = \frac{1}{2}\boldsymbol{v}^t\boldsymbol{R}\boldsymbol{v} - p_x(x - x_0) - p_y(y - y_0) - p_z(z - z_0) \Rightarrow stat. \qquad (4)$$

The internal energy is the expression $\frac{1}{2}\boldsymbol{v}^t\boldsymbol{R}\boldsymbol{v}$. The vector $\boldsymbol{v}^t = (v_x \quad v_y \quad v_z)$ and the matrix $\boldsymbol{R} = diag(q_i \quad q_i \quad q_i)$ show this energy with respect to a single line element *i* .We can write the inner energy as $\frac{1}{2}q_i(v_x^2 + v_y^2 + v_z^2)$, precisely:

$$\begin{aligned} v_x &= x_i - x_c \\ v_y &= y_i - y_c \\ v_z &= z_i - z_c \end{aligned} \qquad \boldsymbol{R} = \begin{bmatrix} q_i & 0 & 0 \\ & q_i & 0 \\ sym. & & q_i \end{bmatrix} \tag{5}$$

The chamber of a pneumatic structure has a volume *V*, which is made by an internal pressure p_i. The product from internal pressure and volume is a part of the total energy Π: a given volume V_0 leads directly to a specific internal pressure p_i : hence the total energy for the Formfinding of a pneumatic chamber is $\prod = \frac{1}{2}\boldsymbol{v}^t\boldsymbol{R}\boldsymbol{v} - p_x(x - x_0) - p_y(y - y_0) - p_z(z - z_0) - p_i(V - V_0) \Rightarrow stat.$ The derivation of the total energy to the unknown coordinates and to the unknown internal pressure ends up with

$$\begin{aligned} \frac{\partial \prod}{\partial x} &= \sum_{i=1}^{m}(x_i - x_c)q_i - p_x - p_i\frac{\partial V}{\partial x} &= 0 \\ \frac{\partial \prod}{\partial y} &= \sum_{i=1}^{m}(y_i - y_c)q_i - p_y - p_i\frac{\partial V}{\partial y} &= 0 \\ \frac{\partial \prod}{\partial z} &= \sum_{i=1}^{m}(z_i - z_c)q_i - p_z - p_i\frac{\partial V}{\partial z} &= 0 \\ \frac{\partial \prod}{\partial p_i} &= V - V_0 &= 0 \end{aligned} \tag{6}$$

In the system (6) the internal pressure p_i can be seen as a so-called Lagrange multiplier. The fourth row in (6) shows, that our boundary condition $V = V_0$ is obtained by the derivation of the energy to this Lagrange multiplier. The vector $(\frac{\partial V}{\partial x} \quad \frac{\partial V}{\partial y} \quad \frac{\partial V}{\partial z})$ describes the normal direction in the point *(x, y, z)* and the size is the according area. By a set of given force-densities for all elements and a given volume V_0 we end up with a pre-stressed and of course balanced pneumatic system with a volume V_0 and an internal pressure p_i.

Each additional chamber leads to an additional volume and Lagrange multiplier, which allows to calculate multi-chambered structures (see Figure 4).

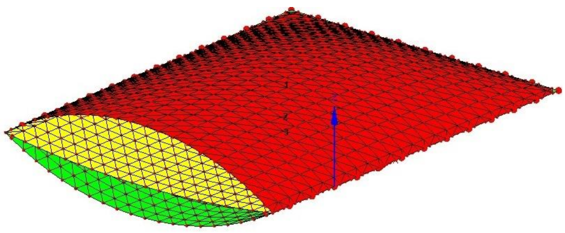

Figure 4: 2 chambers with 3 layers.

If the internal pressure p_i is given or set by user row number 4 of formula (6) is dropped off. The former unknown Lagrange multiplier p_i becomes a known external load.

$$\begin{aligned} \frac{\partial \Pi}{\partial x} &= \sum_{i=1}^{m} (x_i - x_c) q_i = p_x + p_i \frac{\partial V}{\partial x} \\ \frac{\partial \Pi}{\partial x} &= \sum_{i=1}^{m} (y_i - y_c) q_i = p_y + p_i \frac{\partial V}{\partial y} \\ \frac{\partial \Pi}{\partial x} &= \sum_{i=1}^{m} (z_i - z_c) q_i = p_z + p_i \frac{\partial V}{\partial z} \end{aligned} \quad (7)$$

The force-densities *q* and the internal pressure p_i are not independent from each other. This can be simple noted in equation (7). In case of no external loads p_x, p_y, p_z the force densities q_i and the internal pressure p_i are proportional to each other. If we double the prestress we end up with the doubled internal pressure with an unchanged geometry.

Sometimes the membranes are fixed for example to bending stiff beam elements and now we have a situation that the boundary line is not fixed totally. The flexibility of the boundary line should be considered already in the Formfinding stage (also very important for ETFE-cushions). The procedure works as follows. We perform a so-called ‘mixed’ Formfinding, where the boundary lines are defined with all its mechanical properties and for the membrane, we still use the ‘desired’ stress values as always. Now the boundary line deflects but the membrane stresses are perfect with respect to the ‘desired’ input values.

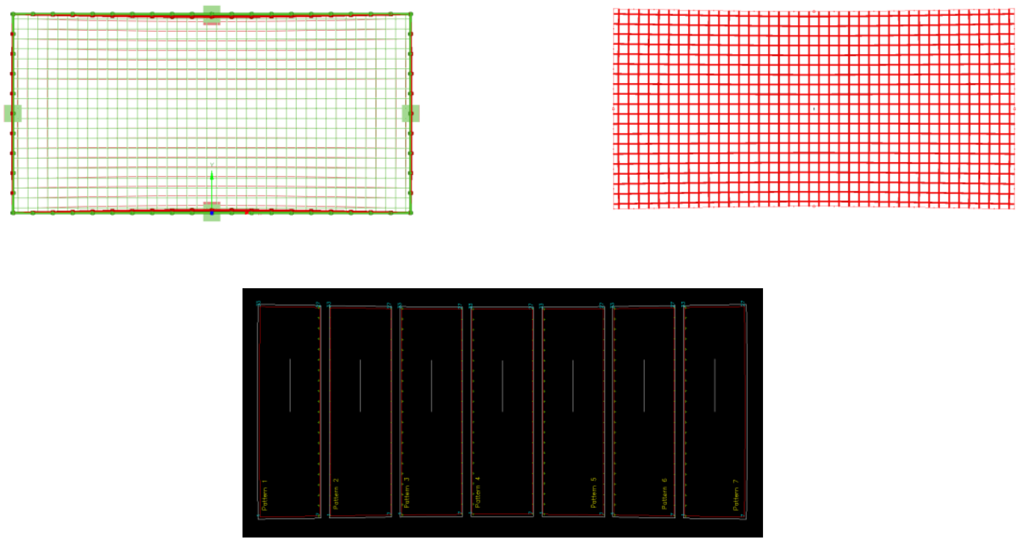

Figure 5: Deflected boundary with uniform stress and cutting patterns

The boundary deflects but the desired prestress in the membrane is maintained. The biggest deflection of the outer beam is as expected in the middle. Therefore, the patterns are smaller in this area.

Very often the membrane system and the primary steel structure are calculated in separated systems by loading the steel elements with the reaction forces of the pneumatic membrane which was fixed at its boundaries in a first calculation step. This procedure is wrong with respect to Formfinding and Statics. Separation is only allowed if the deflections are very small and this is never the case for membrane structures. In case of Statics it is only a first - very imprecise (and expensive) estimation. Users should always calculate with computer models consisting of primary structure and membrane in one holistic model.

1.2 Model generation with geometrically defined surfaces

As already mentioned geometrically defined surfaces must be pneumatically feasible. In this case a Formfinding procedure is not needed. After materialization (=definition of the material properties) a static calculation with a specific internal pressure should not change the geometry significantly (by neglecting elastic deformations). If we find significant geometrical differences the geometry was not perfect with respect to pneumatic feasible forms. The example below seems to be perfect as we have a cylinder and 2 quarter spheres. But the combination of both disturbs the stress-distribution. The lines where the spheres are connected to the cylinder are the problem as the cylinder has the stress $p \cdot R$ and the sphere stress $\frac{1}{2} p \cdot R$. In order to check the differences, we simply perform a static calculation with the internal pressure.

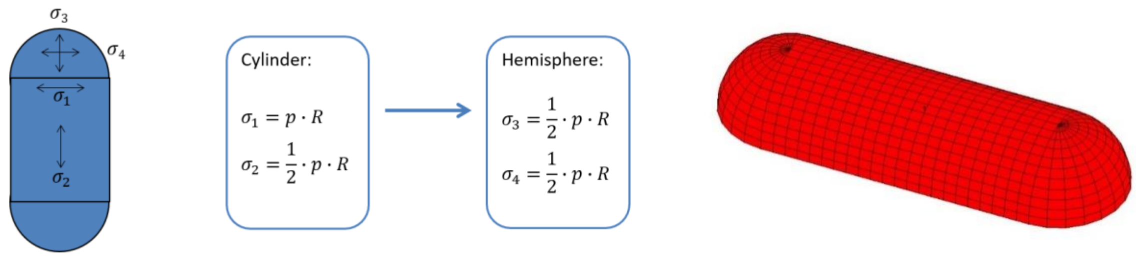

Figure 6: Geometrically defined volume surfaces and stresses

We find the geometrical form under internal pressure more or less unchanged in this example.

1.3 Formfinding with sliding cables

If additional reinforcement cables in case of big wind loads are needed, we perform a Formfinding procedure for those cables in order to find their optimal positions. In Figure 7 we see the cable net in a flat situation.

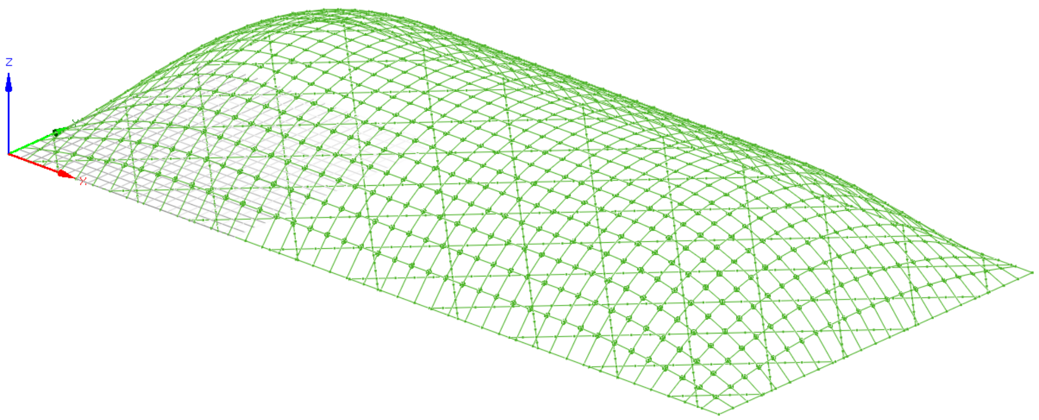

Figure 7: Start situation with a flat cable mesh

The Formfinding procedures calculates the coordinates of the intersection points of the cables onto the surface - which is assumed as fixed – under the constraints that all cable forces are the same. The result is shown in Figure 8.

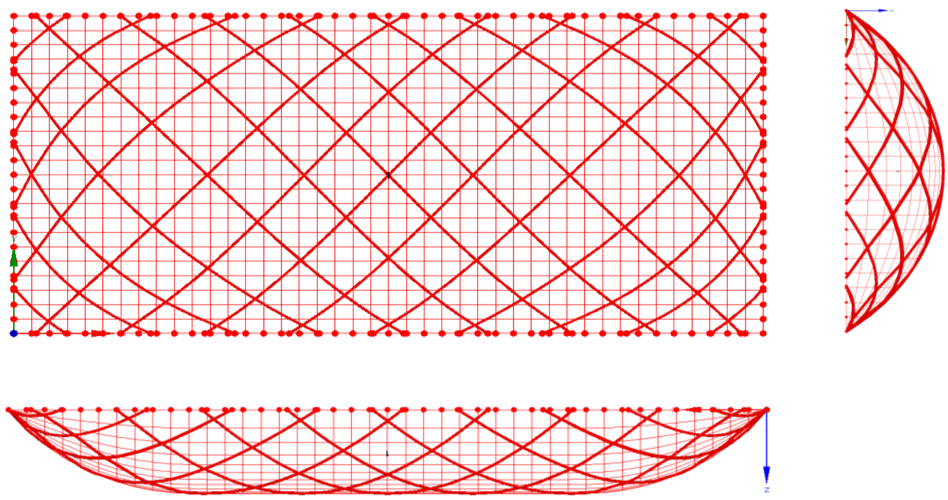

Figure 8: Sliding cables with identical forces after Formfinding (top-, side- and front view)

2 Statics

A static calculation for membranes is geometrically nonlinear. We need material properties for all elements and its nondeformed geometry. The nondeformed geometry of a cable element for instance is the unstressed length of this cable. Next, we need the external loads and the internal pressure or volume information. After the Formfinding-procedure a geometry is available, and Statics can be performed.

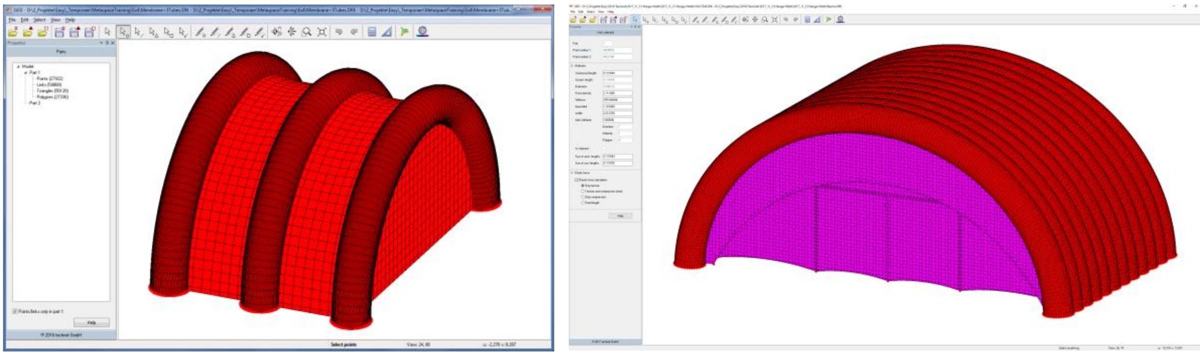

Figure 9: Pneumatic tube systems combined with mechanically stressed membranes

2.1 Statics with Formfinding models

When a usual Formfinding procedure was made also prestress values for all elements exist. We can define material properties for the membranes and then we can calculate the unstressed 'lengths' (= non-deformed geometry) of all elements as we have prestress values from the Formfinding result. Usually the first load-case in statics to be calculated should be 'internal operation pressure'. The result of this calculation must be identical with the Formfinding result as we 'shortened' the membrane elements in this way.

2.2 Statics with geometrically defined models

When a geometrical Formfinding was made, prestress values are usually not available or at least these values do not balance the structure in general. Here we recommend the following procedure:

Define the material properties. The unstressed geometry cannot be calculated by prestress values; therefore, we simply set the stressed lengths to be the unstressed lengths. Now after the load case 'internal operation pressure' we end up with a different geometry. The geometrical differences should be small in this case.

2.3 Theoretical background

2.3.1 Membrane Elements

We extend the form-finding theory by introducing the constitutive equations for the membrane elements to the system (1). Now the force-densities q from the form-finding are unknowns and they belong to the material equations.

$$\begin{bmatrix} \sigma_u \\ \sigma_u \\ \tau \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & 0 \\ & m_{22} & 0 \\ sym. & & m_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_u \\ \varepsilon_v \\ \Delta\gamma \end{bmatrix} \tag{8}$$

We must consider, that the membrane axial-stress in *u*- or *v*- direction can be expressed as $\sigma_u = \frac{S_u}{b_u}$ and $\sigma_v = \frac{S_v}{b_v}$. b_u and b_v are the widths of the *u*- and *v*-lines. The force-densities q can be introduced now as: $S_u = q_u l_u$ and $S_v = q_v l_v$. The strains in *u*- and *v*-direction can be written as follows:

$\varepsilon_u = \frac{l_u - l_{u0}}{l_{u0}}$ and $\varepsilon_v = \frac{l_v - l_{v0}}{l_{v0}}$. The angle difference $\Delta\gamma = \gamma - \gamma_0$ is needed for the shear-stress calculation. γ is the angle between u and *v*-direction; γ_0 refers to the 'non-deformed start-situation' without any shear-stress.

The geometrical compatibility has to be considered as follows:

$l_i == \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2}$ and $\gamma = arccos(\frac{l_u * l_v}{l_u l_v})$, in which $(l_u * l_v)$ means the inner (scalar-) product between u and *v*-direction.

The shear-stress calculation is guaranteed also for a continuous membrane by the fact that the shear angle is between the non-deformed *u*- and *v*-direction of the material [4].

2.3.2 General calculation modes

For static calculations we recommend 4 calculation modes:

1. Given internal pressure p (snow)
2. Given volume V (water)
3. Given product $p{\cdot}V$ (Boyle-Mariotte, for example wind)
4. Given product $\frac{p{\cdot}V}{T}$ (General gas equation, consideration of temperature)

$$\begin{aligned} \frac{\partial \Pi}{\partial x} &= \frac{1}{2}\frac{\partial(v^t R v)}{\partial x} - p_x - \frac{\partial V}{\partial x} p_i &= 0 \\ \frac{\partial \Pi}{\partial y} &= \frac{1}{2}\frac{\partial(v^t R v)}{\partial y} - p_y - \frac{\partial V}{\partial y} p_i &= 0 \\ \frac{\partial \Pi}{\partial z} &= \frac{1}{2}\frac{\partial(v^t R v)}{\partial z} - p_z - \frac{\partial V}{\partial z} p_i &= 0 \\ \frac{\partial \Pi}{\partial p_i} &= V - \frac{(p_{abs} \cdot V)_0}{(p_{atm} + p_i)} &= 0 \end{aligned} \tag{9}$$

Mode 1 is very simple, and we showed the principle in equation (7).

Mode 2 is standard case where row 4 is $V\text{-}V_0 = 0$, see equation (6).

Mode 3 (consideration of gas-laws) enables the realistic behavior of the internal pressure. This mode is important in case of e.g. fast wind gusts. Here the pump systems cannot update the inner pressure in the short time. We can see it as a closed system and by considering the temperature as constant we get the gas law of Boyle and Mariotte $p{\cdot}V{=}const$ in this case. Only if the gas law is fulfilled the membrane stresses get the correct size. Equation (9) refers to mode 3, here the constant value $(p_{abs}{\cdot}V)_0$ is the given product and row 4 of (9) has to be fulfilled in iterations where the unknown internal pressure pi is adapted.

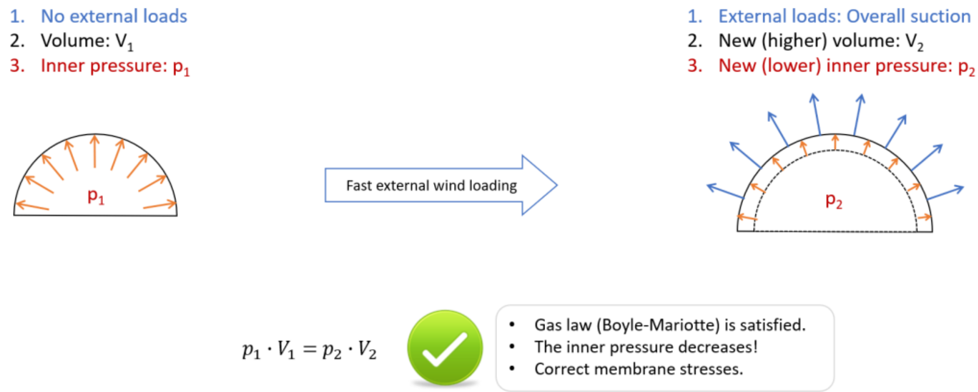

Figure 10: The physical principles of the gas law (mode 3)

On the left-hand side of Figure 10 we see an air hall with an absolute inner pressure p_1 and a volume V_1. The absolute inner pressure is the sum of the over pressure in the air hall and the atmospheric pressure. On the right-hand side, the structure is loaded by an overall wind suction. By considering the Boyle-Mariotte gas law we end up in this case with a higher volume and a lower inner pressure.

Mode 4 considers also the temperature, the principle itself is the same as mode 3 with minor modifications.

By using these modes most cases are covered. The modes can be used e.g. as follow:

1. An air hall under snow-loading (a specific internal pressure is set to resist the snow-loads)
2. A membrane filled with an incompressible fluid (water-bag) and
3. A pneumatic cushion loaded by a fast wind-gust; here, the gas law ($p{\cdot}V{=}const$) is valid.
4. A pneumatic cushion loaded by a fast wind-gust; here, the gas law ($\frac{p{\cdot}V}{T}{=}const$) is valid.

2.3.3 Non-conservative loads

It is to mention that the internal pressure effect is always perpendicular to the deformed geometry. These loads are called non-conservative as all wind loads. In order to get correct results software packages should consider these effects.

Figure 11: Wind load case with different load zones (left) and bending moments in steel ring in a holistic model (right)

2.3.4 Statics with cables as reinforcements

If the membrane stresses are too big, membranes can be combined with cable nets. We distinguish between cables which are fixed to the membrane and "free" sliding cables. These different cases have to be considered in the statico calculation by different calculation strategies. The interaction of membrane and sliding cables has to be modeled correctly. In the cable net (see Figure 12) it is important to receive constant forces in the cables between the crossing points.

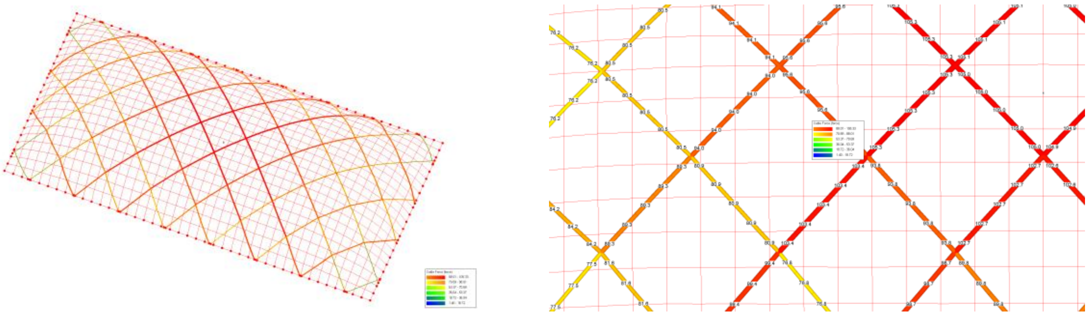

Figure 12: Sliding cable net partially fixed on air hall surface with loops

In Figure 13 the sliding cable is completely free and therefore it is detached from the membrane.

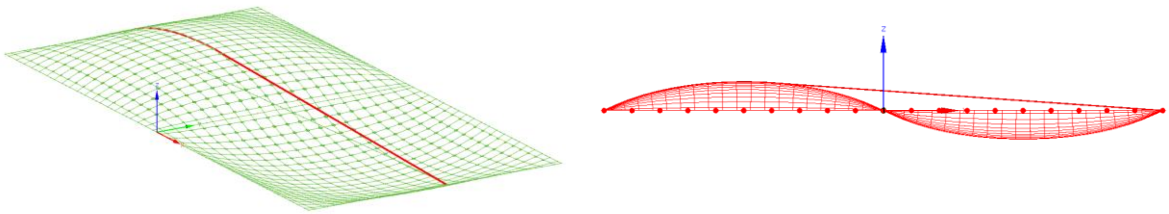

Figure 13: Totally free sliding cable (perspective view and side view)

In Figure 14 we show a situation where an air hall is loaded by wind pressure. The free sliding cables (green lines on right side) are lifted.

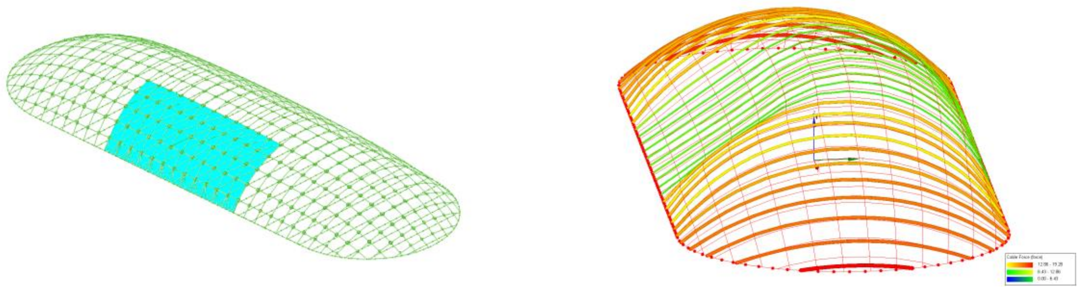

Figure 14: Wind pressure load zone (left), totally free sliding cables detached from the membrane (right)

3 Cutting pattern generation

The cutting patterning is an essential part of the engineering process for pneumatic structures. In the past it was very cost- and time-intensive if the waste of material was to be minimized. Nowadays efficient or even automatic patterning tools are existing where those problems are solved.

3.1 Manual patterning

The first step in the manual patterning procedure is the definition of seam lines. The seam lines can be either geodesic lines or plane cuts defined by starting and ending points. Another possibility is to define so called semi geodesic lines, these are lines with a starting and ending point and a control point in between. If at least 2 lines are defined a flat strip can be generated and its width can be determined.

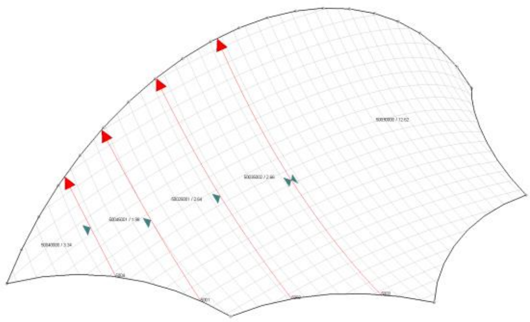

Figure 15: Geodesic seam lines in 3d model

After the determination of the seam lines a flattening method must be executed. The manual patterning is with respect to width optimization and mass production a time-consuming procedure. Therefore, very often automatic methods are used.

3.2 Automatic patterning

In order to optimize the widths of patterns automatically we need optimization variables. The widths of the patterns depend on the position of the seam-lines (mainly geodesic lines). In order to achieve appropriate widths for all patterns the position of those seam lines must be modified until the desired widths of the patterns are reached. Therefore, the position of the geodesic lines must be changed during the iteration process.

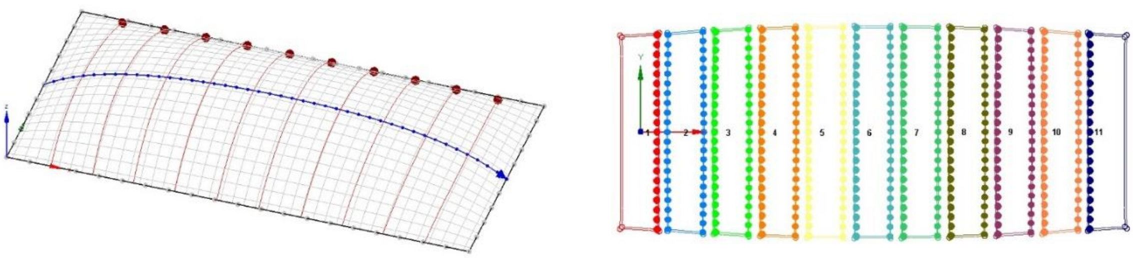

Figure 16: Cushion with automatic created geodesic lines and flattened patterns with identical widths

In case of a non-regular geometry guide lines can be used, if the cutting lines do not intersect themselves, see Figure 17. If not, the seam line definition has to be made manually line by line.

Figure 17: Non-regular geometry with guide lines and patterns

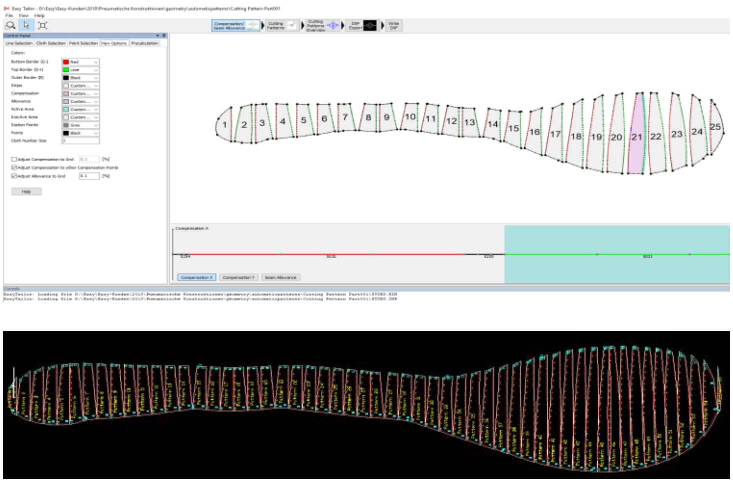

Figure 18: Air hall with automatic created geodesic lines

The cutting pattern layout of air halls should be width optimised but also satisfy the aesthetic sensations. Hierarchical cuts should be possible, this means that the whole surface can be devided in subsurfaces and those parts are patterned independently. In Figure 19 the whole air hall surface is separated in 3 parts and individually patterned (Figure 20). For each individual part automatic pattern procedures were used.

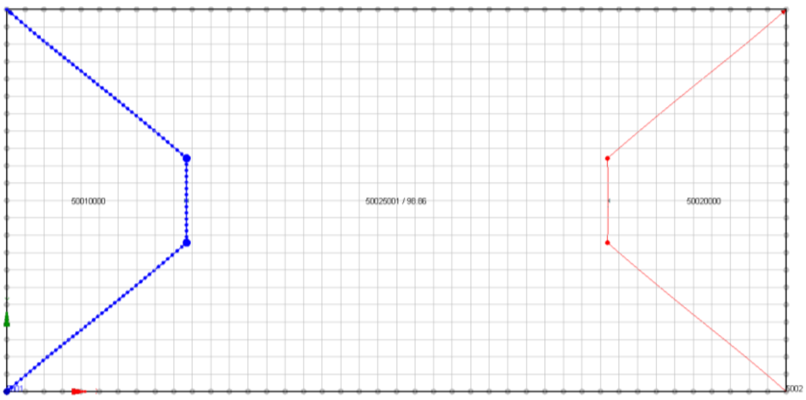

Figure 19: Separation of air hall surface by cutting lines

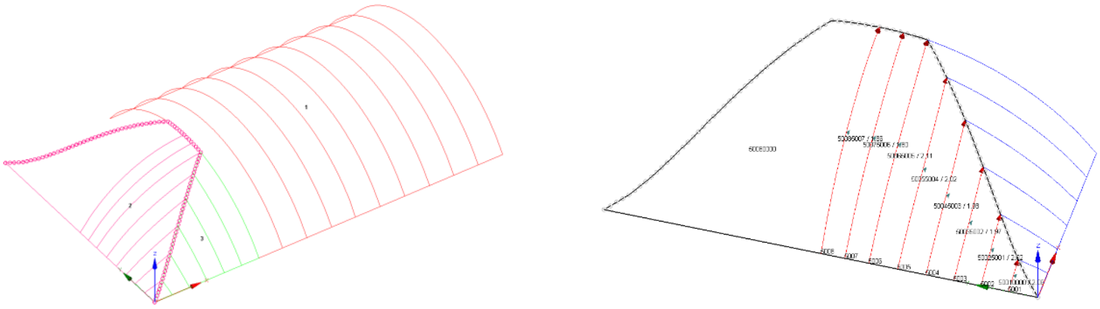

Figure 20: Compliance with geometrical boundary conditions

In Figure 20 we can see that the seam lines of different parts are meeting in one point. Therefore one part can be patterned automatically, the second one has to be done manually.

4 References

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