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On the static calculation of biogas containers with radial and parallel cutting patterns

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Abstract

Nowadays, radial cutting patterns are common for biogas tanks above a certain construction height. The radial cutting patterns are a production challenge, especially at the polar caps. In order to avoid production difficulties, it is desirable to build plants with parallel cutting patterns. In our paper the results of a radial cutting pattern are compared and discussed with the results of a parallel cutting pattern.

Our model deals with the two membrane envelopes of a double membrane system. Together with the inner membrane (gas membrane), an outer membrane forms a chamber, the so-called air support space. Together with the silo walls and the surface of the substrate, the gas membrane forms a second chamber, the so-called gas space.

Different scenarios, e.g. the situation under internal operating pressure and the situation under a gust of wind were simulated. In the case of 'fast' loads (wind), the gas law applies. In the case of rapidly occurring loads, it must be investigated whether the outer shell and the gas membrane touch each other. The contact problem can be considered in our calculation model.

We have applied the internal pressure perpendicular to the surface for each deformed state. We also carried the wind loads with the deformations in the iterations. The material properties are defined by warp-, weft stiffness and the so-called transverse and shear stiffness in order to simulate a realistic behavior in the radial or parallel directions.

Keywords: Pneumatic structures, Biogas plants, Lightweight structures, Formfinding, Statics, Patterning, Gas law, Contact problem, Hybrid structures, Membranes, Foils, Force density, Optimization

1 Introduction

The relationships to the calculation of spheres under internal pressure from isotropic material have been known for a long time. The stress s in the surface is the same in all points and depends in a simple way on the internal pressure p and radius R (s=p*R). A sphere is therefore a regular surface that can be formed pneumatically (such as a cylinder or torus). This simple fact is made considerably more difficult by the fact that textile fabrics do not have isotropic but orthotropic material behavior and are subject not only to internal pressure loads but also, for example, to snow and wind loads. Furthermore, in the simplest case, the structures to be investigated are not spheres, but parts of spheres (spherical calottes), or no regular surfaces anymore, if one does not have a circular, but a polygonal edge. The membrane design is simple in the case of the circular boundary. One simply takes a sphere section. In the case of the polygonal boundary, however, the geometry must be determined in a form-finding process. The result must have a geometry that represents an equilibrium figure under internal pressure loads. These facts are relevant for the determination of the cutting patterns. As with any other surface, any cutting variants and thus different material directions are possible. Static calculations must ensure that the maximum membrane stresses are not exceeded. In the case of the general spherical shape, which was determined by a form-finding process, the material directions must be considered in the form-finding process as the above-mentioned simple relationship s=p*R is not anymore valid in case of orthotropic material. It follows a short description of 'usual' biogas storage systems, so that the formation of computer models becomes understandable.

2 Schematic sketch

Biogas storage systems generally have 2 membrane covers. The outer shell and the so-called gas membrane. The air volume between the outer and inner shell is called the air support space. It is always under pressure. The volume under the gas membrane forms the so-called gas space. There are different pressure situations in the gas space. If the pressure in the gas space is 0, the gas membrane lies on the belts that run from a central point to the silo edges. If the gas pressure is maximum, the volume of the gas space is maximum, and the air support space is minimum.

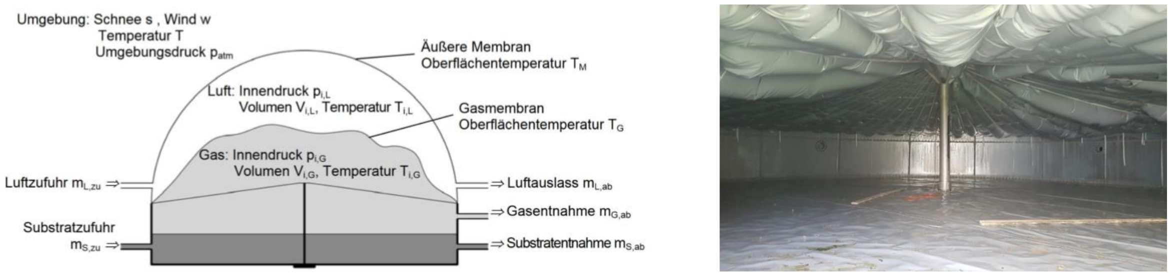


Figure 1: Schematic sketch of a biogas storage system (left), No gas pressure in the biogas-silo, gas membrane lies on the belts (right)

In the static calculation, the external loads must be safely transferred regardless of the situation in the gas and air support space.

Before we come to the calculation results for different variants of cutting patterns, we want to deepen the background of Formfinding and Statics of chambered membranes.

3 Formfinding

The theory of the Formfinding of pneumatic chambers has its basics in the well-known Force-Density Method ([1], [2] and [3]). The Force-Density Method creates a linear system of equations for the form-finding procedure by defining the ratio between Force *S* and stressed length *l* to be known. Hereby the nonlinear equations of the equilibrium change to a linear system.

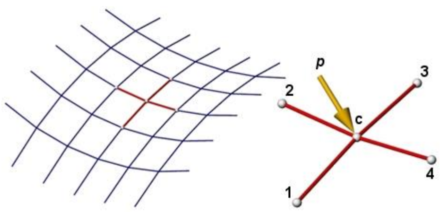


Figure 2: Four cables in point C

In order to clarify these facts *Fig. 1* shows a point *C* which is connected by cables to 4 points (*1*,*2*,*3*,*4*). The nonlinear equations of the equilibrium in the point *C* are as follows, where the external load vector can be expressed $\boldsymbol{p}^t = (p_x \quad p_y \quad p_z)$.

$$
\begin{array}{lllllll}
(x_c - x_1)\dfrac{S_1}{l_1} + & (x_c - x_2)\dfrac{S_2}{l_2} + & (x_c - x_3)\dfrac{S_3}{l_3} + & (x_c - x_4)\dfrac{S_4}{l_4} & = & p_x \\
(y_c - y_1)\dfrac{S_1}{l_1} + & (y_c - y_2)\dfrac{S_2}{l_2} + & (y_c - y_3)\dfrac{S_3}{l_3} + & (y_c - y_4)\dfrac{S_4}{l_4} & = & p_y \\
(z_c - z_1)\dfrac{S_1}{l_1} + & (z_c - z_2)\dfrac{S_2}{l_2} + & (z_c - z_3)\dfrac{S_3}{l_3} + & (z_c - z_4)\dfrac{S_4}{l_4} & = & p_z
\end{array}
\tag{1}
$$

These equations become linear by assuming known force-densities, e.g. $q_1 = \frac{S_1}{l_1}$, and analogue for q_2, q_3 and q_4. The force-density equations are as follows:

$$
\begin{array}{lllllll}
(x_c - x_1)q_1 + & (x_c - x_2)q_2 + & (x_c - x_3)q_3 + & (x_c - x_4)q_4 & = & p_x \\
(y_c - y_1)q_1 + & (y_c - y_2)q_2 + & (y_c - y_3)q_3 + & (y_c - y_4)q_4 & = & p_y \\
(z_c - z_1)q_1 + & (z_c - z_2)q_2 + & (z_c - z_3)q_3 + & (z_c - z_4)q_4 & = & p_z
\end{array}
\tag{2}
$$

The coordinates of the point *C* are the solution of these linear equations. In the following step we want to write the system above by considering m neighbors in the point *C*:

$$
\begin{array}{lll}
\sum_{i=1}^{m}(x_i - x_c)q_i - p_x & = & 0 \\
\sum_{i=1}^{m}(y_i - y_c)q_i - p_y & = & 0 \\
\sum_{i=1}^{m}(z_i - z_c)q_i - p_z & = & 0
\end{array}
\tag{3}
$$

The energy which belongs to the system (1) can be written as (see also [4], [5], [6]).

$$
\prod = \frac{1}{2}\boldsymbol{v}^t\boldsymbol{R}\boldsymbol{v} - p_x(x - x_0) - p_y(y - y_0) - p_z(z - z_0) \Rightarrow stat. \tag{4}
$$

The internal energy is the expression $\frac{1}{2}\boldsymbol{v}^t\boldsymbol{R}\boldsymbol{v}$. The vector $\boldsymbol{v}^t = (v_x \quad v_y \quad v_z)$ and the matrix $\boldsymbol{R} = diag(q_i \quad q_i \quad q_i)$ show this energy with respect to a single line element *i* .We can write the inner energy as $\frac{1}{2}q_i(v_x^2 + v_y^2 + v_z^2)$, precisely:

$$
\begin{array}{lll}
v_x & = & x_i - x_c \\
v_y & = & y_i - y_c \\
v_z & = & z_i - z_c
\end{array}
\qquad
\boldsymbol{R} = \begin{bmatrix} q_i & 0 & 0 \\ & q_i & 0 \\ sym. & & q_i \end{bmatrix}
\tag{5}
$$

The chamber of a pneumatic structure has a volume *V*, which is made by an internal pressure p_i. The product from internal pressure and volume is a part of the total energy Π : a given volume V_0 leads directly to a specific internal pressure p_i : hence the total energy for the Formfinding of a pneumatic chamber is $\prod = \frac{1}{2}\boldsymbol{v}^t\boldsymbol{R}\boldsymbol{v} - p_x(x - x_0) - p_y(y - y_0) - p_z(z - z_0) - p_i(V - V_0) \Rightarrow stat.$ The derivation of the total energy to the unknown coordinates and to the unknown internal pressure ends up with

$$
\begin{aligned}
\frac{\partial \prod}{\partial x} &= \sum_{i=1}^{m} (x_i - x_c) q_i - p_x - p_i \frac{\partial V}{\partial x} &= 0 \\
\frac{\partial \prod}{\partial y} &= \sum_{i=1}^{m} (y_i - y_c) q_i - p_y - p_i \frac{\partial V}{\partial y} &= 0 \\
\frac{\partial \prod}{\partial z} &= \sum_{i=1}^{m} (z_i - z_c) q_i - p_z - p_i \frac{\partial V}{\partial z} &= 0 \\
\frac{\partial \prod}{\partial p_i} &= V - V_0 &= 0
\end{aligned}
\tag{6}
$$

In the system (6) the internal pressure p_i can be seen as a so-called Lagrange multiplier. The fourth column in (6) shows, that our boundary condition $V = V_0$ is obtained by the derivation of the energy to this Lagrange multiplier. The vector $(\frac{\partial V}{\partial x} \;\; \frac{\partial V}{\partial y} \;\; \frac{\partial V}{\partial z})$ describes the normal direction in the point *(x, y, z)* and the size is the according area. By a set of given force-densities for all elements and also a given volume V_0 we end up with a pre-stressed and of course balanced pneumatic system with a volume V_0 and an internal pressure p_i.

4 Statics

A static calculation for membranes is geometrically nonlinear. We need material properties for all elements and its nondeformed geometry. The nondeformed geometry of a cable element for instance is the unstressed length of this cable. Next, we need the external loads and the internal pressure or volume information. After the Formfinding-procedure a geometry is available, and Statics can be performed.

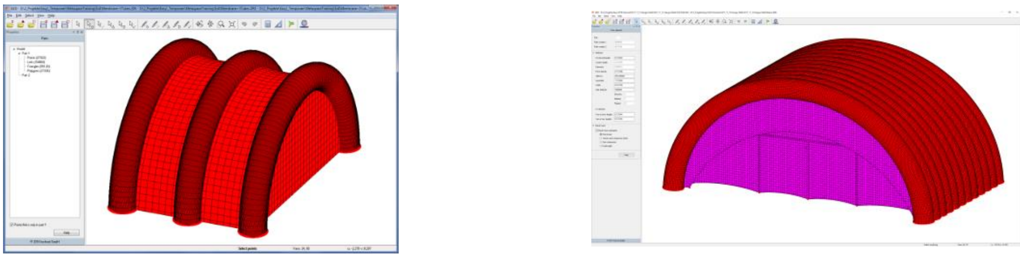


Figure 3: Pneumatic tube systems combined with mechanically stressed membranes

4.1 Statics with Formfinding models

When a usual Formfinding procedure was made also prestress values for all elements exist. We can define material properties for the membranes and then we can calculate the unstressed 'lengths' (= non-deformed geometry) of all elements as we have prestress values from the Formfinding result. Usually the first load-case in statics to be calculated should be 'internal operation pressure'. The result of this calculation must be identical with the Formfinding result as we 'shortened' the membrane elements in this way.

4.2 Statics with geometrically defined models

When a geometrical Formfinding was made, prestress values are usually not available or at least these values do not balance the structure in general. Here we recommend the following procedure:

Define the material properties. The unstressed geometry cannot be calculated by prestress values; therefore, we simply set the stressed lengths to be the unstressed lengths. Now after the load case 'internal operation pressure' we end up with a different geometry. The geometrical differences should be small in this case. If the geometrically defined form was 'pneumatic feasible' the differences are only caused by the elastic deformations in the 'load case' prestress.

4.3 Theoretical background

We extend the form-finding theory by introducing the constitutive equations for the membrane elements to the system (1). Now the force-densities q from the form-finding are unknowns and they belong to the material equations.

$$\begin{bmatrix}\sigma_u\\ \sigma_u\\ \tau\end{bmatrix}=\begin{bmatrix}m_{11} & m_{12} & 0\\ & m_{22} & 0\\ sym. & & m_{33}\end{bmatrix}\begin{bmatrix}\varepsilon_u\\ \varepsilon_v\\ \Delta\gamma\end{bmatrix} \tag{7}$$

We must consider, that the membrane axial-stress in *u*- or *v*- direction can be expressed as $\sigma_u = \frac{S_u}{b_u}$ and $\sigma_v = \frac{S_v}{b_v}$. b_u and b_v are the widths of the *u*- and *v*-lines. The force-densities q can be introduced now as: $S_u = q_u l_u$ and $S_v = q_v l_v$. The strains in *u*- and *v*-direction can be written as follows: $\varepsilon_u = \frac{l_u - l_{u0}}{l_{u0}}$ and $\varepsilon_v = \frac{l_v - l_{v0}}{l_{v0}}$. The angle difference $\Delta\gamma = \gamma - \gamma_0$ is needed for the shear-stress calculation. γ is the angle between u and *v*-direction; $\gamma_0$refers to the 'non-deformed start-situation' without any shear-stress.

The geometrical compatibility has to be considered as follows:

$l_i == \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2}$ and $\gamma = arccos(\frac{l_u * l_v}{l_u l_v})$, in which $(l_u * l_v)$ means the inner (scalar-) product between *u* and *v*-direction.

The shear-stress calculation is guaranteed also for a continuous membrane by the fact that the shear angle is between the non-deformed *u*- and *v*-direction of the material [4].

For static calculations we recommend 4 calculation modes:

1. Given internal pressure p (snow)
2. Given volume V (water)
3. Given product $p{\cdot}V$ (Boyle-Mariotte, for example wind)
4. Given product $\frac{p\cdot V}{T}$ (General gas equation, consideration of temperature)

$$\begin{aligned}
\frac{\partial\Pi}{\partial x} &= \frac{1}{2}\frac{\partial(v^t R v)}{\partial x} - p_x - \frac{\partial V}{\partial x}p_i &= 0\\
\frac{\partial\Pi}{\partial y} &= \frac{1}{2}\frac{\partial(v^t R v)}{\partial y} - p_y - \frac{\partial V}{\partial y}p_i &= 0\\
\frac{\partial\Pi}{\partial z} &= \frac{1}{2}\frac{\partial(v^t R v)}{\partial z} - p_z - \frac{\partial V}{\partial z}p_i &= 0\\
\frac{\partial\Pi}{\partial p_i} &= V - V_0 &= 0
\end{aligned} \tag{8}$$

Mode 1: The internal pressure p_i is no unknown. It is only adapted to the right direction and size during the iterations in the nonlinear calculation.

Mode 2 is standard case for liquid filled membranes. Here we assume that a given Volume V_0 is valid as fluids are assumed to be incompressible.

Mode 3 and Mode 4 (consideration of gas-laws) enables the realistic behavior of the internal pressure. This mode is important in case of e.g. fast wind gusts. Here the pump systems cannot update the inner pressure in the short time. We can see it as a closed system and by considering the temperature as constant we get the gas law of Boyle and Mariotte $p{\cdot}V{=}const$ in this case. Only if the gas law is fulfilled the membrane stresses get the correct size. Equation (8) refers to mode 3, here the constant value $(p_{abs}{\cdot}V)_0$ is the given product and row 4 of (8) has to be fulfilled in iterations where the unknown internal pressure pi is adapted.

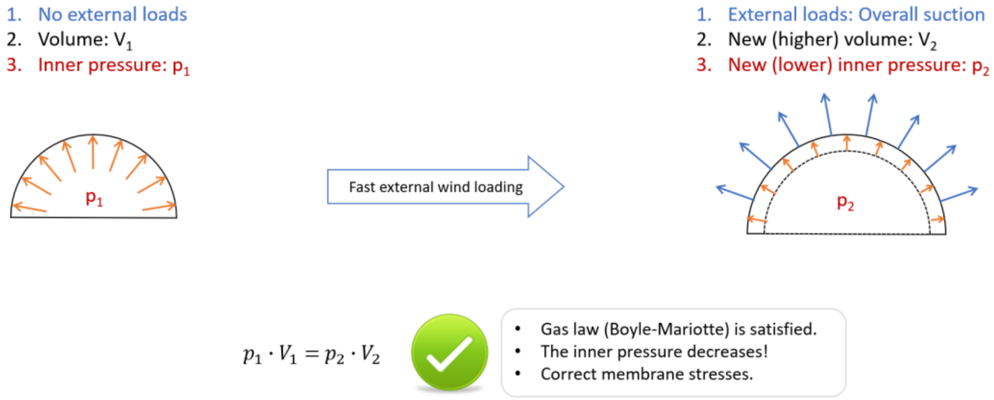


Figure 4: The physical principles of the gas law (mode 3)

On the left-hand side of Figure 4 we see an air hall with an absolute inner pressure p_1 and a volume V_1. The absolute inner pressure is the sum of the over pressure in the air hall and the atmospheric pressure. On the right-hand side, the structure is loaded by an overall wind suction. By considering the Boyle-Mariotte gas law we end up in this case with a higher volume and a lower inner pressure.

Mode 4 considers also the temperature, the principle itself is the same as mode 3 with minor modifications.

By using these modes most cases are covered. The modes can be used e.g. as follow:

1. An air hall under snow-loading (a specific internal pressure is set to resist the snow-loads).
2. A membrane filled with an incompressible fluid (water-bag).
3. A pneumatic cushion loaded by a fast wind-gust; here, the gas law ($p{\cdot}V{=}const$) is valid.
4. A pneumatic cushion loaded by a fast wind-gust; here, the gas law ($\frac{p{\cdot}V}{T}{=}const$) is valid.

It is to mention that the internal pressure effect is always perpendicular to the deformed geometry. These loads are called non-conservative as all wind loads. In order to get correct results software packages should consider these effects.

5 Investigation of modelling approaches

Nowadays, radial cutting patterns are common for biogas tanks above a certain construction height. The radial cutting patterns are a production challenge, especially at the polar caps.

Figure 5: Radial (left) and parallel (right) cutting patterns

In order to avoid production difficulties, it is desirable to build plants with parallel cutting patterns. The polar caps are now side caps, but only with the half side. Those side caps are usually made also from membrane material with 'rotated' material directions.'

5.1 Computer model

The implementation of the biogas storage systems in our software system is described below. In the case of radial cutting patterns (as shown in Figure Figure 5 (left)), the outer shell is modelled with a radial net. The same applies to the gas membrane. In our computer model 2 chambers (gas space and air support space) are generated by surface elements (triangles) and can be controlled individually. In the case of parallel cutting patterns, regular networks can be used. The following picture shows our model with radial meshes.

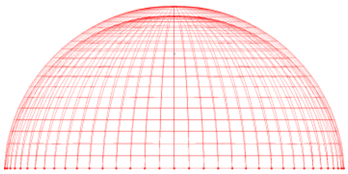


Figure 6: Double Chamber with warp- and weft directions of the textile fabric

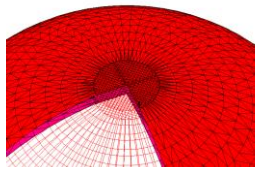


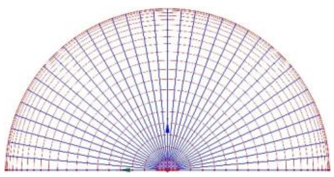
Figure 7: 2 chambers are described by triangular surface elements

5.2 Results of static calculations for radial and parallel cutting patterns

In our calculation we used non-conservative wind loads. We have applied the material properties as follows. It should be noted that the material values at the polar and side caps are doubled.

E_{1000}	E_{2000}	E_{Crimp}	E_{Shear}
800	500	200	50
1600	1000	400	100

Table 1: Membrane material properties



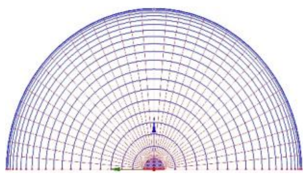


Figure 8: Figure 9: Parallel patterns (warp and weft directions) (gas and outer membrane) with 2 side caps

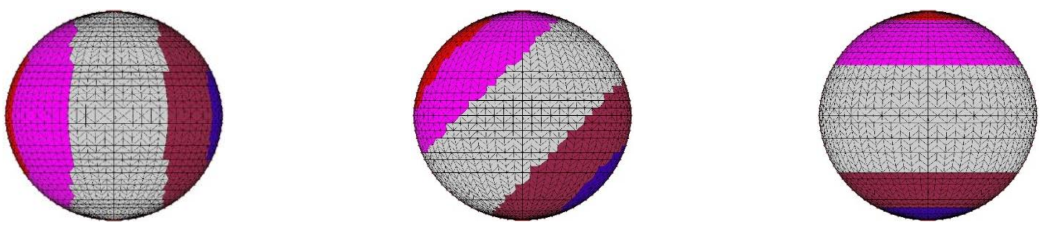


Figure 10: Wind load zone under 3 different wind directions (0°, 45° und 90°).

We have essentially calculated 6 different load cases for different situations. Different filling levels of the gas space were assumed. The 6 load cases were as follows:

- Self-weight and internal pressure.
- Self-weight and increased internal pressure.
- Self-weight, internal pressure and wind 0°.
- Self-weight, internal pressure and wind 45°.
- Self-weight, internal pressure and wind 90°.
- Self-weight, internal pressure and snow.

6 Contact

Contact between outer and inner membrane was considered in case of wind loads; contact happens especially when the gas-membrane is fully stressed with maximum gas volume. The calculation considering contact is important for the right size of the volumes of the air- and gas-space volume and so for the internal pressures in case of gas-law calculations.

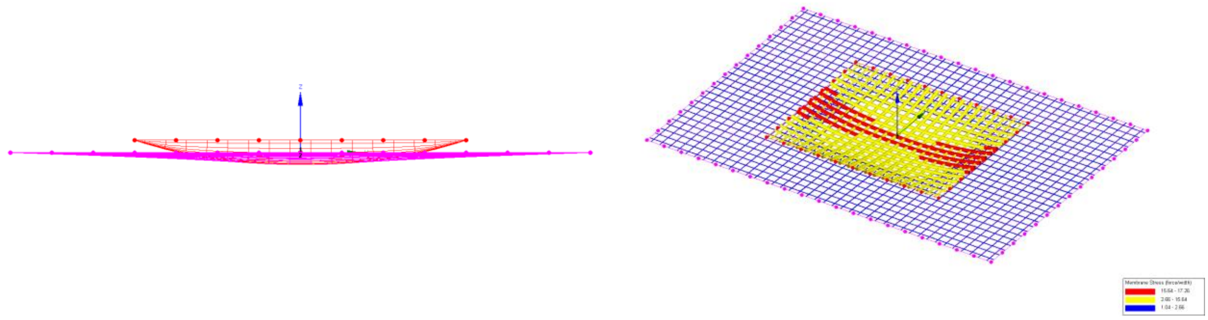


Figure 11: 2 membrane surfaces in contact. Side view (Left), perspective view (right)

7 Conclusion

The differences between the radial and parallel cutting patterns in relation to the size of the maximum membrane stresses were very small. This means that the parallel cutting patterns, which are much easier to produce, can be used in the future.

8 References

[1] Ströbel, D., Singer, P., Holl, J. „Analytical Formfinding", International Journal of Space Structures, Volume: 31 issue: 1, page(s): 52-61, March 1, 2016

[2] Holl, J., Ströbel, D., Singer, P. "Formfinding and Statical Analysis of Cable Nets with flexible covers", *VI International Conference on Textile Composites and Inflatable Structures STRUCTURAL MEMBRANES 2013*, Munich, Germany.

[3] Singer, P. (1995), „Die Berechnung von Minimalflächen, Seifenblasen, Membrane und Pneus aus geodätischer Sicht", *Dissertationsschrift*, *DGK Reihe C*, Nr. 448, 1995.

[4] Ströbel, D. (1997), „Die Anwendung der Ausgleichungsrechnung auf elastomechanische Systeme", *Dissertationsschrift*, *DGK, Reihe C*, Nr. 478.

[5] Ströbel, D. Singer, P, Holl, J: Holistic Calculation of (Multi)-Chambered ETFE-Cushions, *Tensinet*, *TensiNet Istanbul*, 2013.